INVESTIGATION 5-B

Testing Hooke's Law

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting
- Communicating results

Problem

What relationship exists between the force applied to a spring and its extension?

Equipment

- retort stand and C-clamp
- weight hanger and accompanying set of masses
- coil spring
- ring clamp
- metre stick

CAUTION Wear protective eye goggles during this investigation.

Procedure

1. Clamp the retort stand firmly to the desk.
2. Attach the ring clamp close to the top of the retort stand.
3. Hang the spring by one end from the ring clamp.
4. Prepare a data table with the headings: Mass on hanger, m(kg); Applied force, F(N); Height of hanger above desk, h(m); and Extension of spring, x(m).
5. Attach the weight holder and measure its distance above the desktop. Record this value in the first row of the table. This value will be your equilibrium value, h_o, at which you will assign the value of zero to the extension of the spring, x. Put these values in the first line of your table.
6. To create an applied force, add a mass to the weight holder. Wait for the spring to come to rest and measure the height of the weight holder above the desk. Record these values in the table.
7. Complete the second row in the table by calculating the value of the applied force (weight of the mass) and the extension of the spring ($x = h_o - h$).
8. Continue by adding more masses until you have at least five sets of data. Make sure that you do not overextend the spring.

Analyze and Conclude

1. Draw a graph of the applied force versus the extension of the spring. **Note:** Normally, you would put the independent variable (in this case, the applied force) on the x-axis and the dependent variable (in this case, the extension of the spring) on the y-axis. However, the mathematics will be simplified in this case by reversing the position of the variables.
2. Draw a smooth curve through the data points.
3. Describe the curve and write the equation for the curve.
4. State the relationship between the applied force and the extension. This relationship is known as "Hooke's law."
5. When the spring is at rest, what is the relationship between the applied force and the force exerted on the mass by the spring? This force is usually referred to as the "restoring force." Restate the spring relationship in terms of the restoring force of the spring.
6. By finding the area under the graph between the origin and the point of maximum extension, determine the amount of energy stored in the spring.
7. Write an equation for the energy stored in the spring when the slope of the graph is k and the extension is x.
8. Devise and carry out an experiment to determine whether a similar relationship exists for the bending of a metre stick. Obtain your teacher's approval before carrying out the experiment.

Hooke's Law

Investigation 5-B illustrated **Hooke's law**, which states that the amount of extension or compression of a spring varies directly with the applied force. A graphical illustration of this law for an extended spring is shown in Figure 5.9.

Since the data produce a straight line, the equation can be written in the form $y = mx + b$, where m is the slope and b is the y-intercept. The slope of the line describing the properties of a spring, called the **spring constant**, is symbolized by k and has units of newtons/metre. Each spring has its own constant that describes the amount of force that is necessary to stretch (or compress) the spring a given amount. In your investigation, you were directed to assign the reference or zero position of your spring as the position of the spring with no applied force. As a result, x was zero when F was zero. This choice is the accepted convention for working with springs, and it makes the y-intercept equal to zero because the line on the graph passes through the origin. This relationship leads to the mathematical form of Hooke's law (which is summarized in the following box): $F_a = kx$, where F_a is the magnitude of the applied force, x is the magnitude of the extension or compression, and k is the spring constant.

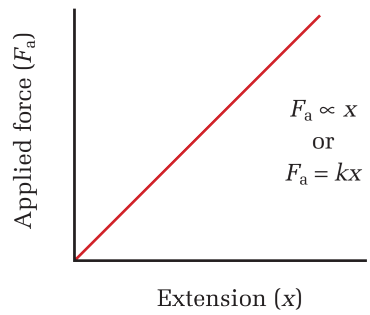

Figure 5.9 The applied force varies directly with the extension of a spring.

HOOKE'S LAW

The applied force is directly proportional to the extension or compression of a spring.

$$F_a = kx$$

Quantity	Symbol	SI unit
applied force	F_a	N (newtons)
spring constant	k	$\frac{\text{N}}{\text{m}}$ (newtons per metre)
amount of extension or compression of the spring	x	m (metres)

Unit Analysis

newtons = $\left(\frac{\text{newtons}}{\text{metre}}\right)$(metre) $\text{N} = \frac{\text{N}}{\cancel{\text{m}}}\cancel{\text{m}} = \text{N}$

According to Newton's third law of motion, the force exerted by the object that is applying the force to the spring is equal and opposite to the force that the spring exerts on that object. The force exerted by the spring is called the **restoring force**. Often, Hooke's law is written in terms of the restoring force of the

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The spring constant is closely related to a quantity called the "modulus of elasticity." This is defined as the stress on the object divided by the strain. Stress is defined as the applied force divided by the cross-sectional area, and the strain is the amount of extension or compression per unit length. This quantity is used to predict how structural components, from aircraft wings to steel beams, will behave when under a given load.

spring: $F_s = -kx$. The negative sign shows that the restoring force is always opposite to the direction of the extension or compression of the spring.

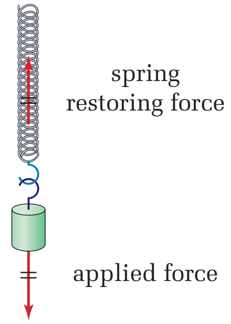

Figure 5.10 The restoring force always opposes the applied force and acts in the direction of the equilibrium position of the spring.

SAMPLE PROBLEM

Hooke's Law in an Archery Bow

A typical compound archery bow requires a force of 133 N to hold an arrow at "full draw" (pulled back 71 cm). Assuming that the bow obeys Hooke's law, what is its spring constant?

Conceptualize the Problem

- When an archer draws a bow, the *applied force* does *work* on the bow, giving it *elastic potential energy*.
- *Hooke's law* applies to this problem.

Identify the Goal

The spring constant, k, of the bow

Identify the Variables and Constants

Known	Unknown
$F_a = 133$ N	k
$x = 71$ cm	

Develop a Strategy

Use Hooke's law (applied force form). $F_a = kx$

Solve for the spring constant. $k = \dfrac{F_a}{x}$

Substitute numerical values and solve.

$$k = \frac{133\ \text{N}}{0.71\ \text{m}}$$

$$k = 187.32\ \frac{\text{N}}{\text{m}}$$

$$k \cong 1.9 \times 10^2\ \frac{\text{N}}{\text{m}}$$

The spring constant of the bow is about $1.9 \times 10^2\ \dfrac{\text{N}}{\text{m}}$.

Validate the Solution

When units are carried through the calculation, the final quantity has units of N/m, which are correct for the spring constant.

PRACTICE PROBLEMS

16. A spring scale is marked from 0 to 50 N. The scale is 9.5 cm long. What is the spring constant of the spring in the scale?

17. A slingshot has an elastic cord tied to a Y-shaped frame. The cord has a spring constant of 1.10×10^3 N/m. A force of 455 N is applied to the cord.

(a) How far does the cord stretch?

(b) What is the restoring force from the spring?

18. The spring in a typical Hooke's law apparatus has a force constant of 1.50 N/m and a maximum extension of 10.0 cm. What is the largest mass that can be placed on the spring without damaging it?

Calculating Elastic Potential Energy

The graph of Hooke's law in Figure 5.9 not only gives information about the forces and extensions for a spring (or any elastic substance), you can also use it to determine the quantity of potential energy stored in the spring. As discussed previously, you can find the amount of work done or energy change by calculating the area under a force-versus-position graph. The Hooke's law graph is such a graph, since extension or compression is simply a displacement. The area under the graph, therefore, is equal to the amount of potential energy stored in the spring, as illustrated in Figure 5.11.

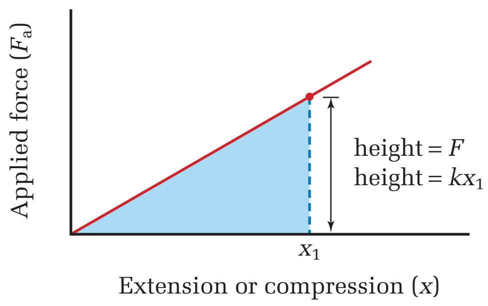

Figure 5.11 The triangular area under the Hooke's law graph gives you the amount of elastic potential energy stored in the spring at any amount of extension.

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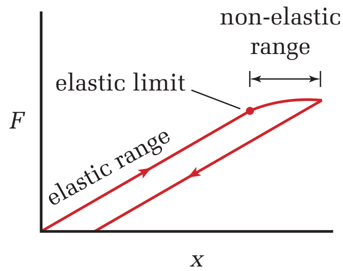
A *perfectly elastic* material will return precisely to its original form after being deformed, such as stretching a spring. No real material is perfectly elastic. Each material has an elastic limit, and when stretched to that limit, will not return to its original shape. The graph below shows that when something reaches its elastic limit, the restoring force does not increase as rapidly as it did in its elastic range.



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Robert Hooke (1635–1703) was one of the most renowned scientists of his time. His studies in elasticity, which resulted in the law being named after him, allowed him to design better balance springs for watches. He also contributed to our understanding of optics and heat. In 1663, he was elected as a Fellow of the Royal Society in London. His studies ranged from the microscopic — he observed and named the cells in cork and investigated the crystal structure of snowflakes — to astronomy — his diagrams of Mars allowed others to measure its rate of rotation. He also proposed the inverse square law for planetary motion. Newton used this relationship in his law of universal gravitation. Hooke felt that he had not been given sufficient credit by Newton for his contribution, and the two men remained antagonistic for the rest of Hooke's life.

As you can see in Figure 5.11, the area under the curve of applied force versus extension of a spring is a triangle. You can use the geometry of the graph to derive an equation for the elastic potential energy stored in a spring.

- Write the equation for the area of a triangle. $A = \frac{1}{2}(\text{base})(\text{height})$
- The elastic potential energy stored in a spring is the area under the curve. $E_e = A$, $E_e = \frac{1}{2}(\text{base})(\text{height})$
- The base of the triangle is the amount of extension of the spring, x_1. $\text{base} = x_1$
- The height of the triangle is the force at an extension of x_1. $\text{height} = F(x_1)$, $F(x_1) = kx_1$, $\text{height} = kx_1$
- Substitute the values into the expression for elastic potential energy. $E_e = \frac{1}{2}(x_1)(kx_1)$, $E_e = \frac{1}{2}kx_1^2$
- The expression is valid for any value of x. $E_e = \frac{1}{2}kx^2$

The equation you just derived applies to any perfectly elastic system and is summarized in the box below.

ELASTIC POTENTIAL ENERGY

The elastic potential energy of a perfectly elastic material is one half the product of the spring constant and the square of the length of extension or compression.

$$E_e = \frac{1}{2}kx^2$$

Quantity	Symbol	SI unit
elastic potential energy	E_e	J (joules)
spring constant	k	$\frac{\text{N}}{\text{m}}$ (newtons per metre)
length of extension or compression	x	m (metres)

Unit Analysis

$$\text{joule} = \frac{\text{newton}}{\text{metre}}\text{metre}^2 \qquad \text{J} = \left(\frac{\text{N}}{\cancel{\text{m}}}\right)\text{m}^{\cancel{2}} = \text{N}\cdot\text{m} = \text{J}$$

SAMPLE PROBLEM

Elastic Potential Energy of a Spring

A spring with spring constant of 75 N/m is resting on a table.

(a) **If the spring is compressed a distance of 28 cm, what is the increase in its potential energy?**

(b) **What force must be applied to hold the spring in this position?**

Conceptualize the Problem

- There is *no change* in the *gravitational potential energy* of the spring.
- The *elastic potential energy* of the spring *increases* as it is compressed.
- *Hooke's law* and the definition of *elastic potential energy* apply to this problem.

Identify the Goal

The elastic potential energy, E_e, stored in the spring
The applied force, F_a, required to compress the spring

Identify the Variables and Constants

Known	Unknown
$k = 75\ \frac{\text{N}}{\text{m}}$	E_e
$x = 0.28$ m	F_a

Develop a Strategy

Apply the equation for elastic potential energy.
Substitute and solve.

$$E_e = \frac{1}{2}kx^2$$
$$E_e = \frac{1}{2}\left(75\ \frac{\text{N}}{\text{m}}\right)(0.28\ \text{m})^2$$
$$E_e = 2.94\ \text{J}$$
$$E_e \cong 2.9\ \text{J}$$

(a) The potential energy of the spring increases by 2.9 J when it is compressed by 28 cm.

Use Hooke's law to calculate the force at 28 cm compression.

$$F_a = kx$$
$$F_a = \left(75\ \frac{\text{N}}{\text{m}}\right)(0.28\ \text{m})$$
$$F_a = 21\ \text{N}$$

(b) A force of 21 N is required to hold the spring in this position.

Validate the Solution

Round the given information to 80 N and 0.3 m and do mental multiplication. The resulting estimated change in elastic potential energy is 3.6 J and the estimated applied force is 24 N. The exact answers are reasonably close to these estimated values. In addition, a unit analysis of the first part yields an answer in N · m or joules, while the second answer is in newtons.

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PRACTICE PROBLEMS

19. An object is hung from a vertical spring, extending it by 24 cm. If the spring constant is 35 N/m, what is the potential energy of the stretched spring?

20. An unruly student pulls an elastic band that has a spring constant of 48 N/m, producing a 2.2 J increase in its potential energy. How far did the student stretch the elastic band?

21. A force of 18 N compresses a spring by 15 cm. By how much does the spring's potential energy change?

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Gravitational potential energy is energy that an object possesses due to its position in a gravitational field. Elastic potential energy is a bit harder to picture. However, when a material is stretched or twisted, its atoms move relative to each other. Since the atoms are held together by electric forces, elastic potential energy is related to the position of an object, such as an atom in an electric field.

Restoring Force and Periodic Motion

The restoring force exerted by a spring always points toward the equilibrium or rest position for that spring. When the spring is extended, the restoring force pulls it back toward its equilibrium position. When the spring is compressed, the restoring force pushes it outward. The nature of this force makes it possible for a spring to undergo a back-and-forth, or oscillating, motion called **periodic motion**. If the restoring force obeys Hooke's law precisely, the periodic motion is called *simple harmonic motion*.

Periodic motion is closely associated with wave motion, a topic that you have studied in previous physics courses. You might recall that a vibrating or oscillating object often creates a wave. You can make the comparison by imagining that you attached a pen to the end of a spring and allowed it to rest on a long sheet of paper. If you extended the spring and then released it, the pen would oscillate back and forth, drawing a line on the paper. If you pulled the paper under the pen at a steady rate while the pen was in motion, you would create an image like the one in Figure 5.12. You probably recognize the figure as having the same shape as the waves that you studied. Many of the terms that you learned in connection with waves also apply to periodic motion.

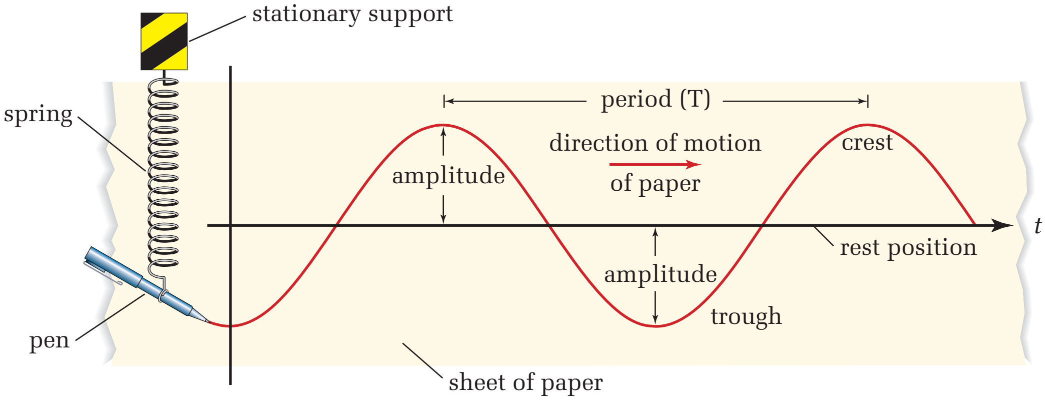

Figure 5.12 This wave shows how the position of the mass at the end of a spring changes with time. Mathematically, this graph is called a sine wave.